Density Stratification and Stability

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Introduction

In most lakes, water properties change from the surface to greater depth, i.e., these lakes show a vertical stratification of their water masses at least for some extended time periods. Heat exchange with the atmosphere and the forming of gradients of dissolved substances controls internal waves and the vertical exchange of water within the lakes. This has decisive impact on the evolution of water quality and, as a consequence, on the community of organisms living in the lake. This article deals with processes contributing to the stratification of lakes and the forming of layers. The most common numerical approaches for the quantitative evaluation of stratification-relevant physical quantities, e.g., electrical conductivity, are included. The final section lists quantities for stability of density stratification and what conclusion can be drawn from them.

Circulation Patterns

Surface temperatures of lakes show a pronounced temperature cycle over the year (Figure 1), in most latitudes. This is a consequence of heat exchange with the atmosphere and the seasonal variation of meteorological parameters, such as incoming solar radiation. The temperatures in the deep water follow the surface temperatures only for the time when the lake is homothermal, as in our example Lake Goitsche, Germany (Figure 1), during winter, from November until April.

Throughout summer, temperatures vary from the surface to the lake bed, and the lake remains stratified. Warmer and less dense water floats on top of colder, denser water (Figure 1). Thus, Lake Goitsche is called stably stratified, as overturning water parcels would require energy. On the contrary, during winter, no density differences obstruct the vertical transport. These seasons are commonly referred to as the stagnation period and the circulation period. Lakes that experience a complete overturn during the year are called holomictic.

During the circulation period, dissolved substances, such as oxygen or nutrients, get distributed over the entire water body (Figure 2). Hence, the circulation pattern is a decisive factor for the evolution of water quality and the biocenosis of the lake. In conclusion, the commonly used classification of lakes is according to their circulation patterns.

- *Holomictic lakes* overturn and homogenize at least once a year.
- In *meromictic lakes*, the deep recirculation does not reach the deepest point of the lake. As a consequence, a chemically different layer of bottom water is formed, the monimolimnion (see below), and remains there for at least 1 year.
- *Amictic lakes* do not experience a deep recirculation. Usually permanently ice-covered lakes are included in this class. Lakes, however, can circulate underneath an ice sheet by external forcing, such as solar radiation that penetrates to the lake bed and geothermal heat flux, or salinity gradients created when ice is forming on a salt lake.
- *Lakes with episodic partial deep water renewal* do not experience a complete overturn. The deep water however is partially replaced in episodic events.

Holomictic lakes are subdivided into classes indicating the frequency of complete overturn.

- *Polymictic* lakes are not deep enough to support a continuous stratification period throughout summer. The entire lake is mixed by sporadic strong wind events over the year or even on a daily basis in response to strong diurnal temperature variation.
- *Dimictic* lakes are handled as the prototype of lakes in moderate to cold climates. A closer look at the lakes, however, reveals that in most cases an ice cover or a great maximum depth is required to guarantee a stratification period during the cold season (see Figure 2). Between ice cover and summer stratification, the lake can be circulated completely in the vertical, the easiest when surface temperatures traverse the temperature of maximum density at 4 °C.
- *Monomictic* lakes possess one circulation period in addition to the stratification period. Many lakes in the temperate climate zone belong into this class, if they do not develop an ice cover during winter. Sometimes such lakes are also referred to as warm monomictic to distinguish them from cold monomictic lakes, which show an ice cover for most of the year and circulate during the short period without ice.
- *Oligomictic* lakes circulate less frequently than once a year, normally at irregular intervals, triggered by extreme weather conditions such as unusually cold winters for the respective location.

As a consequence of the natural variability of the weather conditions between years, the circulation patterns of the lakes also vary. A usually monomictic lake, for example, can show a dimictic circulation pattern when it freezes in an unusually cold winter. As another example, late during the twentieth century, Mono Lake turned meromictic for intermittent periods of 5 or 7 years, respectively, because of inflowing freshwater, but in other years showed a holomictic circulation.

Density Differences and Formation of Layers

Temperature Stratification

Although the surface water is exposed to solar radiation and thermal contact with the atmosphere, the deeper layers are shielded from major sources of heat. Diffusive heat transport on a molecular level is very slow and requires a month for the transport of heat over a vertical distance of 1 m. A much more efficient heat transport can be accomplished by turbulent transport. The energy for the turbulence is mainly supplied by wind stress at the lake surface and transferred via instabilities through friction at the side walls and internal current shear.

Heating a lake over 4 °C at the surface results in a stable stratification. As a consequence, transport of heat to greater depths requires energy. The limited budget of kinetic energy available for mixing limits the depth to which a certain amount of heat can be forwarded over the stratification period. In sufficiently deep lakes, the thermal stratification holds until cooler autumn and winter temperatures permit a deeper circulation. The warm surface water layer is called *epilimnion*, while the colder water layer beneath, which has not been mixed into the epilimnion is called *hypolimnion*. A sharp temperature gradient (*thermocline*) separates both layers (Figure 3).

Epilimnion and atmosphere are in thermal contact and exchange volatile substances with each other. In addition, the epilimnion is recirculated by wind events or periods of lower temperatures during the stratification period. During those periods, dissolved substances are distributed within the epilimnion. On the contrary, the hypolimnion is insulated from exchange with the atmosphere during the stratification period. Transport of dissolved matter across the vertical density gradient of the thermocline usually is small.

In general, wind determines the thickness of the epilimnion, with few exceptions, e.g., where light penetrates beyond the mixing depth because of wind, or where the stratification is determined by

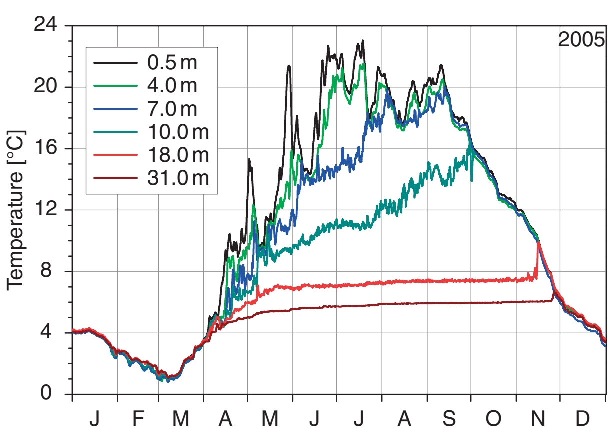

Figure 1 Temperatures (24 h mean) on several depths in Lake Goitsche near Bitterfeld, Germany during the year 2005. Reproduced from Boehrer B and Schultze M (2008) Stratification of lakes. *Reviews in Geophysics*, 46, RG2005, doi:10.1029/2006RG000210, with permission from American Geophysical Union.

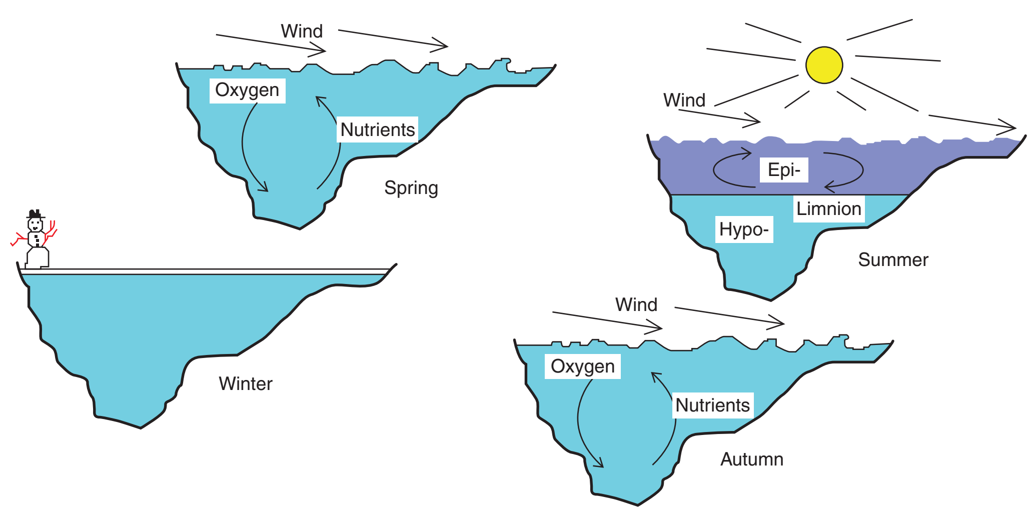

Figure 2 Annual cycle of a dimictic lake with ice cover during winter.

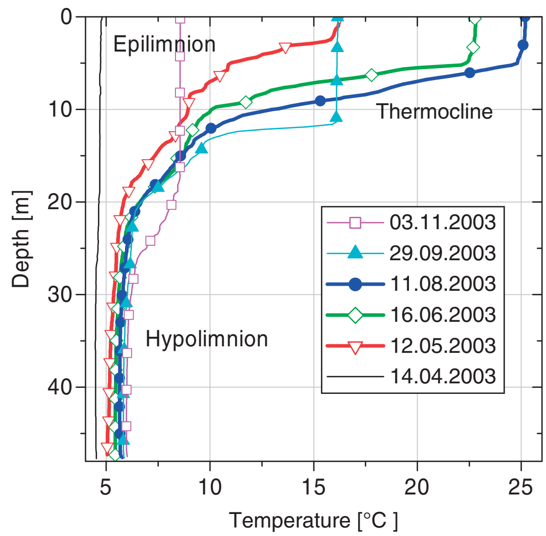

Figure 3 Temperature profiles of Lake Goitsche/Germany in station XN5 in 2003. Symbols are added for every sixteenth data point to distinguish between acquisition dates.

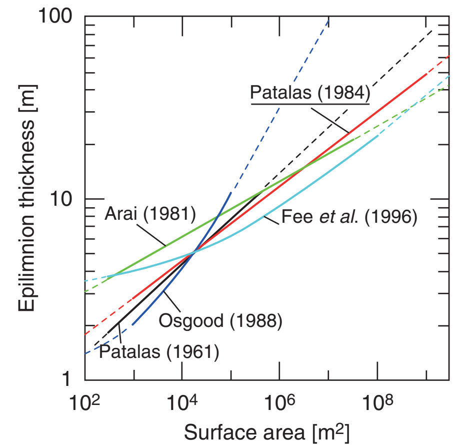

Figure 4 Graphical representation of several approximations of epilimnion thickness z_{epi} versus surface area of the respective lakes. Adapted from Jöhnk KD (2000) 1D hydrodynamische Modelle in der Limnophysik – Turbulenz, Meromixis, Sauerstoff. Habilitationsschrift, Technical University of Darmstadt, Germany.

inflow and water withdrawal (reservoirs). As inferred from Figure 3, the thickness of the epilimnion is not constant over the stratification period. In spring, a thin layer is formed, which gradually thickens over the summer because of the cumulative input of wind energy and diurnal heating and cooling. It takes until autumn, when colder temperatures at the lake surface can erode the stratification. During this later period of thermal stratification, substances dissolved in hypolimnetic waters, such as nutrients, become available in the epilimnion again. Eventually epilimnion and hypolimnion are homogenized.

The epilimnion thickness is a crucial factor for living organisms. Hence limnologists have tried to correlate epilimnion thickness with lake morphometry to achieve an a priori estimate (Figure 4). The most central regression is $h_{epi} = 4.6 \times 10^{-4} A^{0.205}$, which includes the higher energy input from winds over lakes with larger surface area A.

Because of its high gradients, the thermocline forms a special habitat. Organisms controlling their density can position themselves in the strong density gradient. Also, inanimate particles can accumulate on their level of neutral buoyancy and motile organisms dwell in the thermocline to profit from both layers, epilimnion and hypolimnion. As a result, a layer of distinctive properties can form, called *metalimnion*. Especially in nutrient-rich lakes, the decomposition of organic material can deplete oxygen resulting in a so-called metalimnetic oxygen minimum (Figure 5). On the contrary, if light can penetrate to the thermocline and photosynthesis can overcome the oxygen consumption locally, a metalimnetic oxygen maximum occurs.

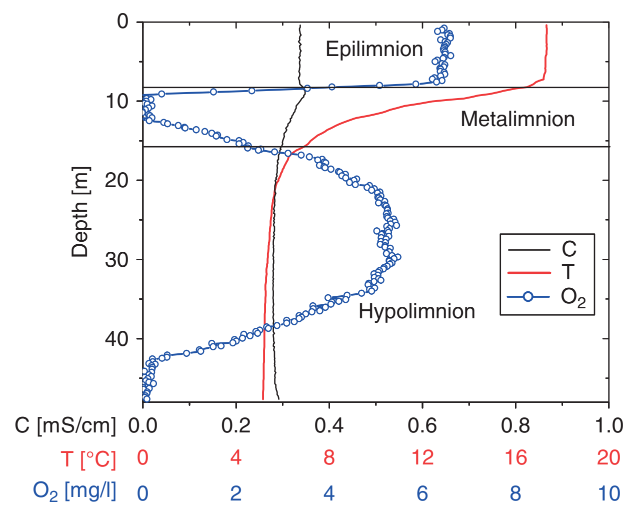

Figure 5 Profiles of temperature (T), (in situ) conductivity (C), and concentration of dissolved oxygen (O_2) from 6 September 2000 in Arendsee/Germany. The boundaries between layers were drawn along the gradients in the oxygen profiles. Oxygen concentration numerically corrected for response time of 7.5 s of the sensor. Adapted from Boehrer and Schultze, 2005, *Handbuch Angewandte Limnologie*, Landsberg: ecomed.

Thermobaric Stratification

Cold water is more compressible than warmer water, in the range of temperatures encountered in lakes. As a consequence, the temperature of maximum density T_{md} decreases as pressure, i.e., depth, increases (by about 0.2 K over 100 m water depth). Hence in cold enough regions, very deep freshwater lakes can show temperature profiles during summer stratification

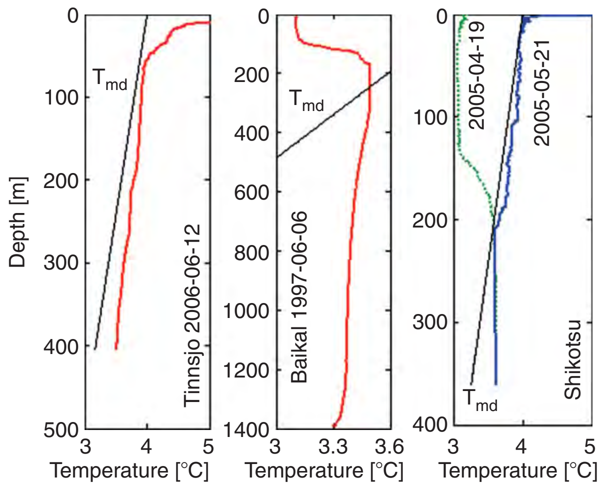

Figure 6 Temperature profiles of thermobarically stratified lakes: Left panel: Tinnsjø, Norway, during (early) summer stratification; Central panel: Lake Baikal, Siberia, Russia, during (late) winter stratification with the vertical transition through T_{md}; Right panel: Lake Shikotsu, Hokkaido, Japan, with the nearly isothermal deep water body below the T_{md} transition. Reproduced from Boehrer B and Schultze M (2008) Stratification of lakes. *Reviews in Geophysics*, 46, RG2005, doi:10.1029/2006RG000210, with permission from American Geophysical Union.

that extend below 4 °C, though limited to the cold side by the T_{md} profile (see Figure 6, left panel). During winter, surface temperature can be lower than T_{md}, while at greater depth the temperatures may be above T_{md} (Figure 6, central panel). Stable density stratification is achieved in these profiles, if above the T_{md} intersection, colder temperatures overlie warmer temperatures, while below this it is the opposite. At the intersection itself, the vertical temperature gradient must disappear. The water body below the T_{md} profile is not directly affected by the annual temperature cycle (Figure 6, right panel).

Nevertheless, observations in such lakes (e.g., Lake Baikal, Russia; Crater Lake, USA; Lake Shikotsu, Japan) show deep waters well supplied with oxygen. This can in part be attributed to the fact that temperatures are low, and productivity and depletion of oxygen happen at a slow rate. However, it is also an indication for a considerable amount of mixing between mixolimnion and the deep water. As a consequence, chemical gradients do not appear in these lakes, and scientists have refrained from calling these lakes meromictic, although a complete overturn does not occur.

Salinity Stratification

A considerable portion of lakes is salty. As there is no compelling boundary, lakes are called salt lakes, if the salt content lies above 3 g in a kilogram of lake water; i.e., 3 g kg^{-1} = 3‰. From this concentration, humans can clearly taste the salt, and ecological consequences become obvious. The salt content in lakes can be as high as 300 g kg^{-1}. However, salinities normally lie below 0.5 g kg^{-1}. Even smaller salinity gradients can determine the circulation of lakes.

Many large salt lakes, e.g., Caspian Sea, Issyk-Kul, Aral Sea, Lake Van, Great Salt Lake, and the Dead Sea, are located in endorheic basins, i.e., areas on the Earth without hydraulic connection to the world ocean at the surface. Salt lakes also occur outside these areas, as solar ponds or basins filled with sea water that lost the connection to the sea. In addition, some inland lakes are fed by saline groundwater.

Salt modifies the properties of lake water. As a quantitative expression, the familiar magnitude of salinity has been transferred from oceanography to limnetic water. One kilogram of ocean water contains about 35 g of salt. Brackish water, i.e., water mixed from sea water and freshwater, shows a similar mix of dissolved substances, while the composition of salts in lakes can greatly deviate from ocean conditions. Consequently, salinity is better replaced by total dissolved substances TDS in the limnetic environment. For quantitative investigations, usually the physical quantity of electrical conductance is suited much better (explained later).

In some lakes, dissolved substances raise the density of the deep waters enough that part of the water column is not recirculated at any time during the annual cycle. The remaining bottom layer, the monimolimnion, can show very different chemical conditions (Figure 7). Such lakes are termed meromictic. Well-known examples are the meromictic lakes of Carinthia, Austria, and Lake Tanganyika, or Lake Malawi. Some small and deep maar lakes as well as natural lakes in southern Norway and Finland are permanently stratified by small concentration differences between mixolimnion and monimolimnion. In addition, deep pit lakes tend to be meromictic.

The monimolimnion is excluded from the gas exchange with the atmosphere over long time periods. Diffusive and turbulent exchange across the chemocline usually is small. As a consequence, anoxia establishes in most cases after sufficient time. Under these chemical conditions, nitrates and sulphates serve as agents for the microbial oxidation of organic material and substances can be produced that would chemically not be stable in the mixolimnion. Permanently exposed to the hydrostatic pressure, gases (CO_2, H_2S, and others) can accumulate in monimolimnion in concentrations far beyond the concentrations encountered in mixolimnia (e.g., Lake Monoun in Cameroun, Africa, Figure 8).

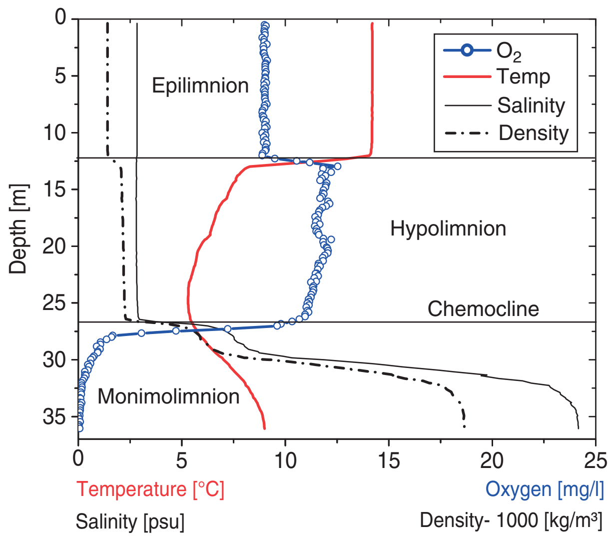

Figure 7 Profiles of temperature, salinity, dissolved oxygen and density from Rassnitzer See in former mining area Merseburg-Ost on 7th October 2003. Oxygen concentrations are numerically corrected for a sensor response time of 7.5s. Adapted from Boehrer and Schultze, 2005, *Handbuch Angewandte Limnologie*, Landsberg: ecomed.

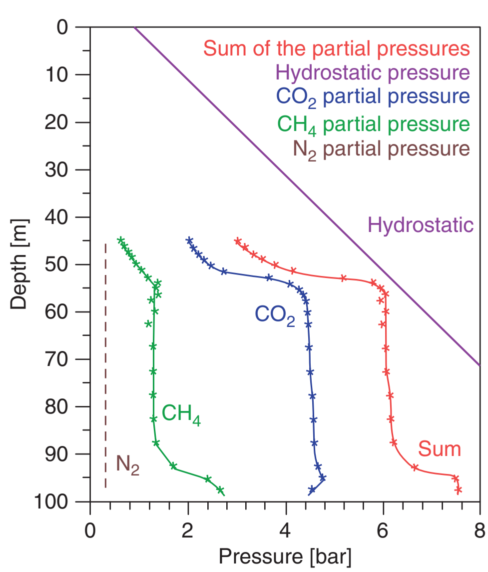

Figure 8 Profiles of partial pressures of dissolved gases in deep water of Lake Monoun, Cameroun, in direct comparison with hydrostatic pressure (solid line). Reproduced from Halbwachs *et al*. (2004) *EOS* 85(30): 281–288, with permission of American Geophysical Union.

In many meromictic lakes, the recirculation of the mixolimnion erodes the monimolimnion leaving a sharp gradient at the end of the circulation period. The transition of all water properties happens within few decimeters from mixolimnetic to monimolimnetic values (see Figure 7). This sharp gradient is called *halocline*, *chemocline*, or *pycnocline*, depending on whether the salinity, chemical, or density gradient is referred to. From observations, cases of intensive colonization with only few different species are known, where some plankton species obviously take advantage of such gradients (e.g., Lago Cadagno in the Swiss Alps, Lake Bolvod, Gek Gel, and Maral Gel).

Processes Forming Gradients of Dissolved Substances

In general, diffusive and turbulent diffusive processes distribute dissolved substances more equally throughout a lake over time. On the contrary, inflows of different concentration of dissolved substances or internal processes can produce gradients within a lake. Freshwaters can flow onto a salt lake or enter the epilimnion during the stratification period and hence induce a difference between epilimnion and hypolimnion. These gradients contribute to the density gradients and if strong enough they may even control the circulation pattern of the lakes. In extreme cases, saline waters have been captured in deep layers of lakes for several thousands of years (e.g., Rorhopfjord, Norway; Salsvatn, Norway; Powell Lake, Canada). Such stable stratifications have been induced on purpose in mine lakes to confine heavy metals in the salty monimolimnion for further treatment (Island Copper Mine Lake, Vancouver Island, BC, Canada).

Also, the opposite case of lakes being exposed to high evaporation can encounter salinity gradients in the water column with the saltier layer above. In the case of Lake Svinsjøen (Norway), salty water from deicing roads has removed the previously present permanent stratification. In another case, when ice is forming on salt lakes the residual water can be highly loaded with salts, while after ice melt, relatively fresh water floats on more saline water (lakes in Antarctica). Also, groundwater inflows can form such gradients of dissolved substances (e.g., Lago Cardagno in Swiss Alps, Rassnitzer See; see Figure 7). Especially, lakes in volcanic areas are known for the continuous recharge of dissolved substances (e.g., Lake Nyos and Lake Kivu in East Africa, Lake Monoun see Figure 8). In particular, the latter case is interesting, as the dissolved gases contribute decisively to the stable density stratification, but they also supply the buoyancy for catastrophic limnic eruptions, in which poisonous gases escape abruptly from a lake with disastrous consequences for living organisms in the area.

Chemical reactions and biological activity in preferred layers can locally change the composition of dissolved substances and impact on the density structure. Although in most lakes at most depths physical transport mechanisms prevail, in some cases the density stratification is controlled by chemical and biological transformations of dissolved material and even control the circulation pattern of lakes. For example, photosynthetically active plankton uses incoming solar radiation for the production of organic material. In addition, allochthonous material is carried into the lake by surface inflows and wind. A portion of the organic material settles on the lake bed. Its decomposition is facilitated by the presence of oxygen or other oxidizing agents and the end products CO_2 and HCO_3^- dissolve in the deep layers of the lake where they contribute to the density. Also iron (and manganese) cycling, calcite precipitation, and sodium sulfate precipitation have been documented to control density in lakes.

In some cases, gradients are strong enough to prevent overturns. These meromictic lakes are classified according to the dominant process sustaining the density difference between mixolimnion and monimolimnion in ectogenically (surface inflow; e.g., Rorhopfjord and Salsvatn in Norway; Powell Lake and Island Copper Mine Lake in Canada, Lower Mystic Lake in the United States), crenogenically (groundwater inflow; e.g., Lago Cardagno, Rassnitzer See, Lake Nyos), or biogenically (or endogenically) meromictic lakes (decomposition of organic material; e.g., Woerther See and Laengssee in Austria, iron cycle: Lakes in Norway, mine lakes, manganese: Lake Nordbytjernet, Norway; calcite: Lake La Cruz, Central Spain; sodium sulfate: Canadian prairie lakes). Also, basin depth and basin shape play an important role in the erosion of a monimolimnion and the formation of deep water renewal. Often monimolimnia are found in well-defined depressions in a lake bed, which are only marginally impacted by basin scale currents in the water body above.

Episodic Partial Deep Water Renewal

A number of lakes do not experience a complete overturn. Episodic events replace parts of the deep water. For example, by cooling, water parcels of high density are formed within the mixolimnion and manage to proceed through the surrounding waters down into the deep water. This process is similar to the deep ocean circulation. Thus, it is not surprising that some of the largest lakes undergo this process. Issyk-Kul (central Asia) recharges its deep water by surface cooling and channelling the cold waters through submerged valleys to the abyss, while in Lake Baikal, wind forcing can push water below the compensation depth so that (temperature-dependent) compression under high pressure raises the density enough compared with surrounding water that its buoyancy becomes negative; the water parcel continues to proceed deeper. Issyk-Kul and Lake Baikal do not show significant chemical gradients and hence are not termed meromictic. On the contrary, Lake Malawi (East Africa) shows an anoxic monimolimnion, and hence is called meromictic, although the deep water formation is similar to that of Issyk-Kul.

In parallel to heat, concentration gradients of dissolved substances also can force deep water renewal in perennially stratified water bodies. Salinity is increased when water is exposed to high evaporation in a side basin (Dead Sea before 1979), or salt accumulates underneath a forming ice cover (Deep Lake, Antarctica). As soon as water parcels become dense enough, they can proceed along slopes into deep parts of the lake.

Quantifying Stability

Temperature

Temperatures recorded in lakes are so-called in situ temperatures. Without any further annotation, temperature data will be understood as such. Nearly all calculations refer to this value, as it is the physically, chemically, and ecologically relevant magnitude. However, if detailed considerations on stability and vertical temperature gradients are envisaged, the reference to potential temperature may be useful. This latter quantity includes the effect of energy required for the expansion, when a water parcel is transferred to atmospheric pressure:

$$\left(\frac{\mathrm{d}T}{\mathrm{d}z}\right)_{\mathrm{ad}} = \frac{g\alpha(T + 273.15)}{c_p} \quad [1]$$

where α is the thermal expansion coefficient for a water parcel of temperature T along the path from depth z to the surface. In lakes where the deep water is close to temperatures of maximum density T_{md}, the thermal expansion coefficient is very small, $\alpha \approx 0$, and, as a consequence, the difference between in situ temperature and potential temperatures is small $\Theta \approx T$. In lakes with warmer deep waters, α can be considered constant. Figure 9 shows a monotonous potential temperature profile in Lake Malawi, Africa, which indicates stable stratification by temperature only.

Salinity, Electrical Conductivity and Electrical Conductance

Many substances in lake water are dissolved as ions. Hence electrical conductivity has been used to

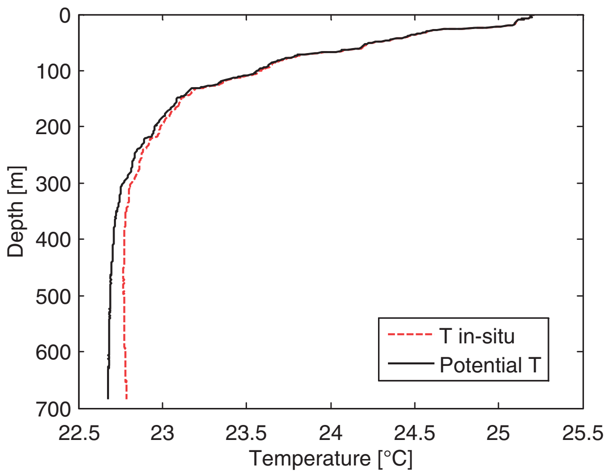

Figure 9 Profiles of (in situ) temperature T and potential temperature near the deepest location of Lake Malawi on 13 September 1997. Reproduced from Boehrer B and Schultze M (2008) Stratification of lakes. *Reviews in Geophysics*, 46, RG2005, doi:10.1029/2006RG000210, with permission from American Geophysical Union.

quantify dissolved substances. For compensation purposes, the temperature dependence of electrical conductivity of a water sample is recorded, while scanning the relevant temperature interval. In most cases, a linear regression $C(T) = aT + b$ is satisfactory to define the conductance, i.e., the electrical conductivity $\kappa_{\mathrm{ref}} = C(T_{\mathrm{ref}}) = aT_{\mathrm{ref}} + b$ at a certain reference temperature T_{ref}. Most commonly, 25 °C is used for the reference:

$$\kappa_{25} = \frac{C(T)}{\alpha_{25}(T - 25°C) + 1} \text{ where } \alpha_{\mathrm{ref}} = (T_{\mathrm{ref}} + b/a)^{-1} \qquad [2]$$

In most surface waters, a value close to $\alpha_{25} = 0.02\ \mathrm{K}^{-1}$ is appropriate. Electrical conductance is used for a bulk measurement of concentrations of ionically dissolved substances, quantifying transports from changes in the conductance profile, and to base density regression curves on.

Oceanography uses electrical conductivity and temperature to calculate salinity in practical salinity units (psu), which gives a good indication for dissolved salt in grams per kilogram for ocean water and brackish water (water mixed from ocean water and fresh water). In limnetic systems, however, the composition of dissolved substances differs from that of the ocean. Even within some lakes, there are pronounced vertical gradients. As a consequence, salinity can only be used with reservation in limnic systems.

Density

As direct density measurements in the field are not accurate enough, indirect methods based on easy to measure temperature and conductivity are implemented. As in most practical applications the difference between in situ and potential temperature is small; we use T for temperature. For lakes of low salinities (<0.6 psu), density can be approximated:

$$\rho = \rho(S, T) = \sum_{i=0}^{6} a_i T^i + S \cdot \sum_{i=0}^{2} b_i T^i \qquad [3]$$

using

$$a_i = [999.8395; 6.7914 \times 10^{-2}; -9.0894 \times 10^{-3}; 1.0171 \times 10^{-4}; -1.2846 \times 10^{-6}; 1.1592 \times 10^{-8}; -5.0125 \times 10^{-11}]$$

$$b_i = [0.8181; -3.85 \times 10^{-3}; 4.96 \times 10^{-5}]$$

In lakes of a composition of dissolved substances similar to the ocean, the so-called UNESCO formula may be applied (e.g., Rassnitzer See in Figure 6), which is applicable for salinities above 2 psu. In cases where salinity cannot be used, calculation of density may directly be based on measurements of temperature and conductivity. For Lake Constance – Obersee, the following formula was proposed, where 20 °C was used as reference temperature for conductance κ_{20}:

$$\rho = \rho_T + \Gamma = 999.8429 + 10^{-3} \times (0.059385\,T^3 - 8.56272\,T^2 + 65.4891\,T) + \Gamma \qquad [4]$$

adding the conductivity contribution in separate

$$\Gamma = \gamma\kappa_{20} \text{ and } \gamma = 0.67 \times 10^{-3}\ \mathrm{kgm^{-3}mS^{-1}cm} \qquad [5]$$

Alternatively, if the dissolved substances are known, e.g., from chemical analysis, density can be calculated by adding the separate contributions:

$$\rho = \rho_T\left(1 + \sum_n \beta_n C_n\right) \qquad [6]$$

where C_n is the concentration of the substance n (g kg^{-1}). A short table of coefficients

$$\beta_n = \frac{1}{\rho}\left(\frac{\partial\rho}{\partial C_n}\right)_{\Theta,p,C_m} \quad m \neq n$$

is given in Table 1. In most limnological applications, density is used for stability considerations. Hence, density refers to potential density, i.e., the density of a certain water parcel under normal atmospheric conditions (1013 hPa). As a consequence of the (small) adiabatic compressibility of water, in situ density increases with pressure, i.e., water depth, by about 5×10^{-10} Pa. This means that at 200 m depth, (potential) density and in situ density differ by about 10^{-3}.

Stability

Stability of a water column derives from the density increase in the vertical. Hence, it is a measure for the

Table 1 Contribution of dissolved or suspended substances to the density of water

Substance	*β_n[(kg/kg)]*
$Ca(HCO_3)_2$	0.807
$Mg(HCO_3)_2$	0.861
$Na(HCO_3)$	0.727
$K(HCO_3)$	0.669
$Fe(HCO_3)_2$	0.838
$NH_4(HCO_3)$	0.462
CO_2	0.273
CH_4	−1.250
Air	−0.090

Modified from Imboden, DM and Wüest A (1995). *Physics and Chemistry of Lakes*, pp. 83–138. Berlin: Springer-Verlag.

potential energy required for vertical excursion and for overturning of water parcels. Stability considerations can be made for an interface in the water column or for the entire stratified water body as a whole. If an energy source is known, the ratio of required and supplied energy yields a nondimensional number.

Differential Quantities The stability of a density stratification is quantified by

$$N^2 = -\frac{g}{\rho}\frac{d\rho}{dz} \qquad [7]$$

where g is the acceleration due to gravity, and z is the vertical coordinate. The magnitude N is also called stability frequency or Brunt–Väisälä frequency (s^{-1}), which indicates the maximum frequency (ω) for internal waves that can propagate in the respective stratification. N^2 indicates how much energy is required to exchange water parcels in the vertical.

As a consequence, chemical gradients can only persist for longer time periods where density gradients limit the vertical transport of dissolved substances (see Figure 10). Lake basin Niemegk of Lake Goitsche (Germany) has been neutralized by introducing buffering river water to the epilimnion. During summer 2000, the vertical transport through the temperature stratification was limited and a chemical gradient in pH could be sustained (Figure 11). However, in winter the temperature stratification vanished, vertical transport was enhanced, and consequently, chemical gradients were removed. Gradients in the pH close to the lake bed were stabilized by increased density because of higher concentration of dissolved substances.

In a stratified water column, a current shear can supply kinetic energy for producing vertical

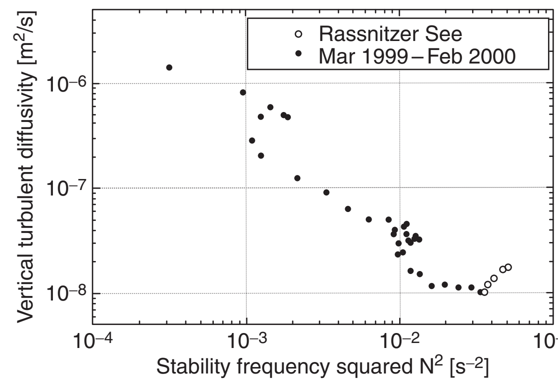

Figure 10 Turbulent diffusive transport of an artificial tracer (SF_6) in the strongly stratified monimolimnion of Rassnitzer See, versus density gradient, $N^2 = -g/\rho\ d\rho/dz$. Adapted from von Rohden and Ilmberger (2001) *Aquatic Sciences* 63: 417–431.

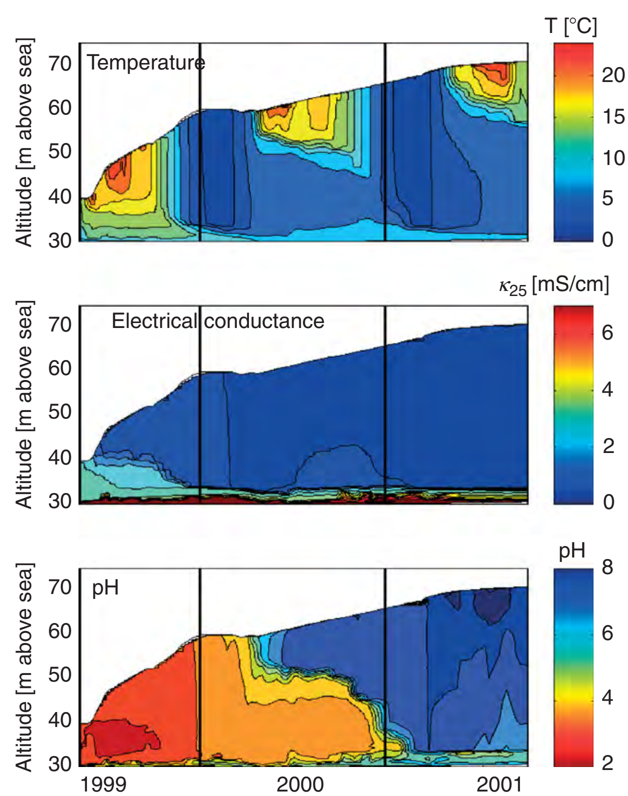

Figure 11 Contour plot of temperature, electrical conductance and pH value versus time and depth in mining Lake Goitsche (station XN3 in Lake basin Niemegk); period of neutralization by flooding with river water; the rising water level is represented by the increasing colored area.

excursions and overturns. A comparison between density gradient and current shear yields the nondimensional gradient Richardson number:

$$Ri = \frac{N^2}{(du/dz)^2} \qquad [8]$$

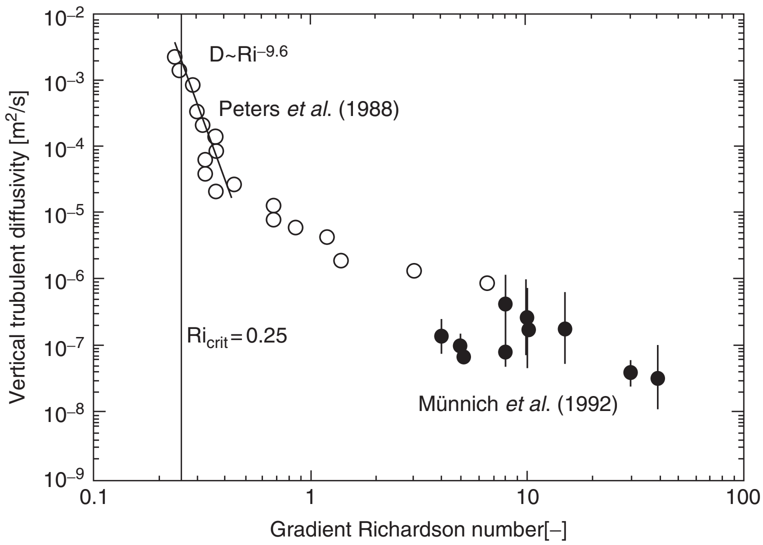

Figure 12 Relation between diapycnal diffusivities and gradient Richardson number. Adapted from Imboden DM and Wüest A (1995) *Physics and Chemistry of Lakes*, pp. 83–138. Berlin: Springer-Verlag.

where $u = u(z)$ represents the horizontal current velocity profile. The critical value of $Ri = 1/4$, when the shear flow supplies enough energy to sustain overturning water parcels, is found by considering the energy balance in the centre of mass frame. As a consequence, diapycnal transports rapidly increase, if Richardson numbers get close to 0.25 or even fall below this critical value (see Figure 12). Although in zones of high shear, e.g., in the bottom boundary layer, supercritical Richardson numbers can be found, they appear only sporadically in the pelagic region of lakes, at least if measured on a vertical scale of meters. The measurements in Figure 12 suggest a correlation between vertical transport coefficients and gradient Richardson number of

$$D = 3 \times 10^{-9} Ri^{-9.6} + 7 \times 10^{-6} Ri^{-1.3} + 1.4 \times 10^{-7} (\mathrm{m^2 s^{-1}}) \quad [9]$$

if the value of molecular diffusivity of heat is included. On the basis of this, vertical diffusivities can be calculated from gradient Richardson number measurements, which are displayed in Figure 13, where gradient Richardson number was measured in pelagic waters over a depth resolution of 3–5 m.

Bulk Quantities For the bulk stability of a stratified water body, various quantities have been proposed, based on potential energy integrated from lake bottom z_b to the surface z_s (e.g., Birge work). We list the definition of Schmidt stability S_t as the most often used reference for the work required for mixing a stratified lake:

$$S_t = \int_{z_b}^{z_s} (z - z_V)(\rho(z) - \bar{\rho}) A(z) \mathrm{d}z \quad [10]$$

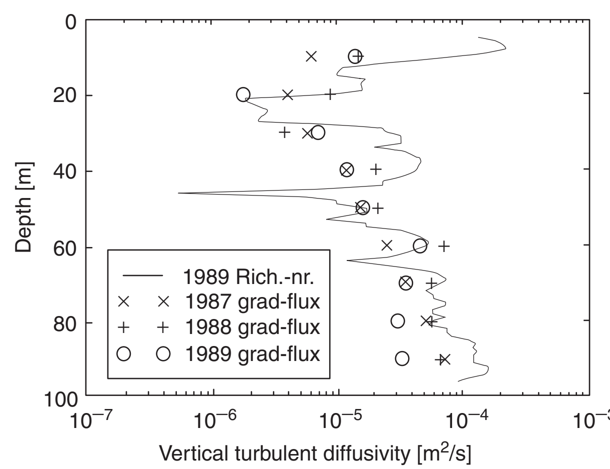

Figure 13 Profiles of vertical transport coefficients in Lake Constance during the stratification period, based on gradient Richardson number measurements in the area of high shear at the Sill of Mainau (solid line), in comparison with results of the gradient flux method for the entire lake based on the evolution of temperature profiles in Überlinger See in years 1987, 1988 or 1989. Adapted from Boehrer *et al.* (2000) *Journal of Geophysical Research* 105(C12): 28,823.

where $z = z_V = \frac{1}{V} \int_{z_b}^{z_s} z A(z) \mathrm{d}z$ is the vertical position of the centre of lake volume V, and $\bar{\rho}$ is the density of the hypothetically homogenized lake.

In a two-layer system, like thermally stratified lakes, it is reasonable to compare the potential energy needed for vertical excursion with the wind stress applied to the surface as done by the Wedderburn number W:

$$W = \frac{g' h_{\mathrm{epi}}^2}{u_*^2 L} \quad [11]$$

where $g' = g\frac{\Delta\rho}{\rho}$, with $\Delta\rho$ representing the density difference between epilimnion and hypolimnion, $u_*^2 = \tau/\rho$ is the friction velocity resulting from the surface stress τ implied by the wind, and L stands for the length of the fetch.

Although the common use of the Wedderburn number is connected to its simplicity, the more sophisticated Lake number $L_N = M_{bc}/\left(z_V \int_A \tau \mathrm{d}A\right)$ compares the wind stress applied to the lake surface with the angular momentum M_{bc} needed for tilting the thermocline, and hence represents the integral counterpart of the Wedderburn number.

Small values of both, W and L_{N}, indicate that wind stress can overcome restoring gravity forces because of density stratification. Under such conditions, upwelling of hypolimnion water is possible and intense mixing of hypolimnion water into the epilimnion can be expected. A typical consequence of ecological importance facilitated by this process is the recharging of nutrients in the epilimnion from the hypolimnion.

Nomenclature

a,a_i,b,b_i	coefficients
A	area, especially surface area of a lake (m^2)
c_p	specific heat ($J\ K^{-1}\ kg^{-1}$)
C	electrical conductivity ($mS\ cm^{-1}$)
C_n	concentration of substance ($g\ kg^{-1}$)
D	vertical turbulent diffusivity ($m^2\ s^{-1}$)
h_{epi}	thickness of epilimnion (m)
g	acceleration due to gravity (m^2/s)
g'	reduced acceleration due to gravity (m^2/s)
L	length of lake or wind fetch (m)
L_N	lake number
M_{bc}	angular momentum (N m)
N	stability frequency (s^{-1})
Ri	gradient Richardson number
$Ri_{\mathrm{crit}} = 0.25$	critical gradient Richardson number
$[\ ]_{\mathrm{ref}},[\ ]_{25}$	at reference temperature, mostly 25°C
S	salinity for fresh water or ocean conditions (psu)
S_{t}	Schmidt stability (kg m)
T	(in situ) temperature (°C)
T_{md}	temperature of maximum density (°C)
T_{ref}	reference temperature (°C)
u	horizontal current velocity ($m\ s^{-1}$)
u_*	friction velocity ($m\ s^{-1}$)
V	lake volume (m^3)
W	Wedderburn number
z	vertical coordinate (m)
z_b,z_s,z_V	vertical coordinate of lake bed, surface, centre of volume (m)
$\alpha = \left(\frac{\partial\rho}{\partial T}\right)_{\mathrm{S,p}}$	thermal expansion (K^{-1})
$\alpha_{\mathrm{ref}},\alpha_{25}$	coefficient at reference temperature, at 25 °C (K^{-1})
β_n	coefficient for specific density contribution of salts
γ	conductivity specific (potential) density contribution ($kg\ m^{-3}\ mS^{-1}\ cm$)
Γ	(potential) density contribution by dissolved substances ($kg\ m^{-3}$)
κ_{ref},κ_{25}	electrical conductance at reference temperature, at 25 °C ($mS\ cm^{-1}$)
ρ	(potential) density ($kg\ m^{-3}$)
$\rho_{\mathrm{in\ situ}}$	(potential) density ($kg\ m^{-3}$)
ρ_{T}	(potential) density of pure water ($kg\ m^{-3}$)
$\bar{\rho}$	reference density ($kg\ m^{-3}$)
Θ	potential temperature (K)
τ	surface stress (Pa)
ω	wave frequency (rad/s)

See also: The Benthic Boundary Layer (in Rivers, Lakes and Reservoirs); Chemical Properties of Water; Currents in Stratified Water Bodies 1: Density-Driven Flows; Currents in Stratified Water Bodies 2: Internal Waves; Currents in Stratified Water Bodies 3: Effects of Rotation; Currents in the Upper Mixed Layer and in Unstratified Water Bodies; Effects of Climate Change on Lakes; Lakes as Ecosystems; Meromictic Lakes; Mixing Dynamics in Lakes Across Climatic Zones; Paleolimnology; Physical Properties of Water; Saline Inland Waters; Salinity; Small-scale Turbulence and Mixing: Energy Fluxes in Stratified Lakes; The Surface Mixed Layer in Lakes and Reservoirs.

Further Reading

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Relevant Websites

http://www.ilec.or.jp/ – International Lake Environment Committe; data on various lakes on Earth.
http://www.ioc.unesco.org/ – Intergovernmental Oceanographic Commission of UNESCO; provides on-line calculator for salinity following the so-called UNESCO formula.
http://www.cwr.uwa.edu.au/ – Centre for Water Research (CWR) at The University of Western Australia. Research Institution focussing of physical limnology.
http://www.eawag.ch/ – Swiss Federal Institute of Aquatic Science and Technology. Institution dealing with water related issues.